

Quantum mechanics differential equations and the de Broglie wavelength.

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(This work is an extract of [4] listed in sec. References.)

Abstract

Quantum mechanics differential equations are based on the de Broglie wavelength assigned to a particle.

This paper presents the effect on quantum mechanics differential equations when replacing the de Broglie wavelength by a relation between the radius and the total energy of a particle. This relation results from a theoretical work of the author about the interaction of charged particles, where particles are modelled as emitting and absorbing continuously fundamental particles with longitudinal and transversal angular momentum. The laws of interaction are mathematically formulated and then proven in that the basic laws of physics (Coulomb, Ampere, Lorentz, Maxwell, Gravitation, bending and interference of light and particles), are deduced from them.

The main effect on quantum mechanics is the change of the pair of canonical conjugated variables that are linked by the uncertainty principle of Heisenberg, namely, to the pairs **energy-space** and **momentum-time**. This has the consequence on relativistic and non relativistic operators in that they are reversed respect to time and space derivation compared with the current theory.

The accordance of the proposed theory with the correspondence principle of the older quantum mechanics theory is proven, in that the time independent differential equation from Schroedinger, deduced from the wave package constructed on the de Broglie wavelength, can be derived from the wave package constructed on the radius-energy relation.

By replacing the de Broglie wavelength with the relation between the radius and the total energy of a particle, the wave-particle duality is eliminated and a new set of differential equations results to describe quantum mechanics.

1 Introduction.

Quantum mechanics differential equations are based on the de Broglie wavelength. In the original work about the interaction of charged particles [4], this particles are represented by a non-local model emitting and absorbing continuously fundamental particles. The energy of a particle is distributed in space from r_o to infinity, where r_o is the radius of the particle. The following relation between the radius r_o and the energy of a charged particle is derived .

$$r_o = \frac{\hbar c}{E} \quad \text{with} \quad E_{tot} = \sqrt{E_o^2 + E_p^2} \quad \text{the relativistic energy.} \quad (1)$$

This relation is used instead of the de Broglie wavelength in order to build wave packages with a Gauss distribution and to derive the corresponding probability differential equations of quantum mechanics. The effects on the uncertainty relations and the main quantum mechanics operators are presented.

2 General considerations.

To make use of the of Fourier-Transformation, the movement of a particle is first described as a sequence of particles represented by a sinus wave, having a wavelength λ equal to $2\pi r_o$. Then the Fourier-Transformation of a wave package of sinus waves with a Gauss shaped amplitude is build.

We have that

$$\lambda = 2\pi r_o = 2\pi \frac{\hbar c}{E_{tot}} \quad \text{with} \quad E_{tot} = \sqrt{E_o^2 + E_p^2} \quad (2)$$

or

$$E_{tot} = E_o + E_{kin} = E_o + E_o \left[\frac{1}{2} \mu - \frac{1 \cdot 1}{2 \cdot 4} \mu^2 + \frac{1 \cdot 1 \cdot 3}{2 \cdot 4 \cdot 6} \mu^3 - \frac{1 \cdot 1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8} \mu^4 + \dots \right] \quad (3)$$

with

$$E_o = m_o c^2 \quad E_p = p c \quad p = \frac{m_o c}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \mu = \frac{E_p^2}{E_o^2} \quad (4)$$

The sinus wave on the x-axis is

$$\xi_x = A e^{i(k_x x - \omega_x t)} \quad \text{with} \quad k_x = \frac{2\pi}{\lambda_x} \quad \text{and} \quad \omega_x = 2\pi \frac{v_x}{\lambda_x} \quad (5)$$

If we now introduce in the expression that $\lambda_x = 2\pi r_{o_x} = 2\pi \hbar c / E_{tot_x}$ we get

$$\xi_x = A \exp \left[i \frac{c}{\hbar} \left(\frac{E_{tot_x}}{c^2} x - \frac{v_x}{c^2} E_{tot_x} t \right) \right] \quad (6)$$

or

$$\xi_x = A \exp \left[i \frac{c}{\hbar} \left(\frac{E_{tot_x}}{c^2} x - p_x t \right) \right] \quad (7)$$

with

$$E_{tot_x} = E_o + E_{kin_x} = m_o c^2 \left(1 - \frac{v_x^2}{c^2} \right)^{-1/2} \quad \text{and} \quad p_x = \frac{v_x}{c^2} E_{tot_x} \quad (8)$$

with E_{kin_x} the relativistic kinetic energy of the particle on the x-axis.

3 The wave package.

We define the Fourier-Transformation of a wave package [1,2]; on the x-axis as

$$\phi_x(x, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \kappa_x(p_x) \exp \left\{ i \frac{c}{\hbar} [m_{tot_x}(p_x) x - p_x t] \right\} dp_x \quad (9)$$

with a Gauss distribution $\kappa_x(p_x)$ on the p_x -axis

$$\kappa_x(p_x) = B \exp \left\{ -\frac{(p_x - p_{x_o})^2}{4(\Delta p_x)^2} \right\} \quad (10)$$

and the dispersion $m_{tot_x} = m_{tot_x}(p_x)$ with

$$m_{tot_x} = \frac{E_{tot_x}}{c^2} \quad m_{tot_x} = m_{tot_x}(p_x) = \frac{1}{c^2} \sqrt{E_o^2 + p_x^2 c^2} \quad \text{and} \quad E_o = m_o c^2 \quad (11)$$

Because of symmetry reasons we also have a wave package

$$\psi_x(x, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \chi_x(m_{tot_x}) \exp \left\{ i \frac{c}{\hbar} [m_{tot_x} x - p_x(m_{tot_x}) t] \right\} dm_{tot_x} \quad (12)$$

with the Gauss distribution on the m_{tot_x} -axis

$$\chi_x(m_{tot_x}) = A \exp \left\{ -\frac{(m_{tot_x} - m_{tot_x_o})^2}{4(\Delta m_{tot_x})^2} \right\} \quad (13)$$

and the dispersion

$$p_x(m_{tot_x}) = c \sqrt{m_{tot_x}^2 - m_o^2} \quad \text{and} \quad m_o = \frac{E_o}{c^2} \quad (14)$$

Note: When deriving the wave-packge with the radius-energy relation, the mass of a particle is considered as concentrated in a sphere with a diameter equal approximately to two times the radius r_o given by the radius energy-relation. This is not according to the approach that let to the radius-energy relation where the mass (energy) of a particle is distributed from r_o to infinity, outside the sphere with radius r_o .

4 Probability differential equations.

In this section the probability differential equations are derived. The differential equations are classified into unrestricted and non-relativistic. Then they are subclassified in groups of general, time or space independent.

4.1 Unrestricted differential equations.

The unrestricted differential equations are valid for the whole range of speed $0 \leq v \leq c$.

We start with the wave package

$$\psi_x(x, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \chi_x(m_{tot_x}) \exp \left\{ i \frac{c}{\hbar} [m_{tot_x} x - p_x(m_{tot_x}) t] \right\} dm_{tot_x} \quad (15)$$

with

$$m_{tot_x} = \frac{E_{tot_x}}{c^2} \quad \text{and} \quad p_x(m_{tot_x}) = c \sqrt{m_{tot_x}^2 - m_o^2} \quad (16)$$

with

$$E_{tot_x} = E_o + E_{kin_x} = \sqrt{E_o^2 + E_{p_x}^2} \quad E_o = m_o c^2 \quad E_{p_x} = p_x c \quad (17)$$

For the unrestricted range of velocities $0 \leq v \leq c$ we have that

$$p_x = \frac{v_x}{c^2} E_{tot_x} \quad (18)$$

and E_{kin_x} represents the kinetic energy for the whole range of speed.

4.1.1 The wave equation.

The wave differential equation we obtain by derivation of ψ_x two times versus t and two times versus x . The results are then connected through

$$p_x = \frac{v_x}{c^2} E_{tot_x} \quad (19)$$

We get

$$\frac{\partial^2}{\partial x^2} \psi_x = \frac{1}{v_x^2} \frac{\partial^2}{\partial t^2} \psi_x \quad (20)$$

For $v_x \rightarrow c$ we have

$$\frac{\partial^2}{\partial x^2} \psi_x(x, t) = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \psi_x(x, t) \quad (21)$$

the well known wave equation

4.1.2 The time independent differential equation.

Time independent differential equations are deduced derivating one time and two times the wave function ψ_x .

a) We derivate the wave function ψ_x one time versus x and get the following time independent differential equation on the x coordinate

$$\frac{\partial}{\partial x} \psi_x = \frac{i}{\hbar c} E_{tot_x} \psi_x = \frac{i}{\hbar c} (E_o + E_{kin_x}) \psi_x \quad (22)$$

E_{kin_x} represents the kinetic energy for the whole range of speed, relativistic and non-relativistic.

b) We derivate the wave function ψ_x two times versus x and get the following time independent differential equation on the x coordinate

$$\frac{\partial^2}{\partial x^2} \psi_x = - \frac{c^2}{\hbar^2} m_{tot_x}^2 \psi_x \quad (23)$$

With

$$m_{tot_x} = \frac{1}{c^2} \sqrt{E_o^2 + E_{p_x}^2} \quad E_o = m_o c^2 \quad \text{and} \quad E_{p_x} = p_x c \quad (24)$$

we get

$$\frac{\partial^2}{\partial x^2} \psi_x = - \frac{1}{\hbar^2 c^2} (E_o^2 + E_{p_x}^2) \psi_x \quad (25)$$

4.1.3 The space independent differential equation.

We derivate the wave function ψ_x two times versus t

$$\frac{\partial^2}{\partial t^2} \psi_x = - \frac{c^2}{\hbar^2} p_x^2 \psi_x \quad (26)$$

and with

$$E_{p_x} = p_x c \quad \text{and} \quad E_p^2 = E_{p_x}^2 + E_{p_y}^2 + E_{p_z}^2 \quad (27)$$

we get

$$- \hbar^2 \frac{\partial^2}{\partial t^2} \psi_x = E_{p_x}^2 \psi_x \quad (28)$$

and for the space

$$- \hbar^2 \Delta_t \psi = E_p^2 \psi \quad (29)$$

with the operator Δ_t defined in sec. 6.

4.2 Non relativistic differential equations

For non relativistic speeds we have that $v \ll c$ and that $E_{kin_x} \approx p^2/(2m_o)$.

4.2.1 General non relativistic differential equation.

The general non relativistic differential equation we obtain by deriving ψ_x two times versus t and one time versus x . The results are then connected through $E_{kin_x} \approx p^2/(2m_o)$. We get

$$i \hbar c \frac{\partial}{\partial x} \psi_x(x, t) \approx \frac{\hbar^2}{2 m_o c^2} \frac{\partial^2}{\partial t^2} \psi_x(x, t) - E_o \psi_x(x, t) \quad \text{with} \quad E_o = m_o c^2 \quad (30)$$

The differential equation with the constant energy E_o describes the movement of a non-accelerated particle in a zero potential energy field. For an accelerated movement in a field with a potential energy $U_x(x, t)$, the energy varies with space and time. This is considered by adding $U_x(x, t)$ to the constant Energy E_o . The result is

$$- i \hbar c \frac{\partial}{\partial x} \psi_x(x, t) \approx \left\{ - \frac{\hbar^2}{2 m_o c^2} \frac{\partial^2}{\partial t^2} + (E_o + U_x) \right\} \psi_x(x, t) \quad (31)$$

Note: If we define the sinus wave on the x-axis (5) as $\xi_x = A e^{-i(k_x x - \omega_x t)}$ the resulting differential equation is

$$i \hbar c \frac{\partial}{\partial x} \psi_x(x, t) \approx \left\{ - \frac{\hbar^2}{2 m_o c^2} \frac{\partial^2}{\partial t^2} + (E_o + U_x) \right\} \psi_x(x, t) \quad (32)$$

Comparing equation (32) with the **General Schrödinger** differential equation, the main difference is that equation (32) derives one time versus space and two times versus time, in other words, time and space are interchanged.

4.2.2 The time independent non relativistic differential equation.

Differential equations are deduced in derivating one time or two times the wave function ψ_x .

a) We derivate the wave function ψ_x one time versus x

$$\frac{\partial}{\partial x} \psi_x = \frac{i}{\hbar c} E_{totx} \psi_x = \frac{i}{\hbar c} (E_o + E_{kinx}) \psi_x \quad (33)$$

For a conservative field $U_x = q_e V_x$ with a total energy E_{G_x} we have

$$E_{G_x} = E_{kinx} + U_x \quad \text{and with} \quad E_{kinx} \approx \frac{1}{2 m_o} p_x^2 \quad (34)$$

we get

$$\left\{ -i \hbar c \frac{\partial}{\partial x} + U(x) \right\} \psi(x) \approx E_x \psi(x) \quad (35)$$

with

$$E_x = E_{G_x} + E_o \quad (36)$$

the Eigenvalue.

b) For the time independent differential equation deduced derivating the wave function ψ_x two times versus x see sec. 7.

4.2.3 Space independent non relativistic differential equation.

We take two times the derivate of the wave function ψ_x versus t

$$\frac{\partial^2}{\partial t^2} \psi_x = -\frac{c^2}{\hbar^2} p_x^2 \psi_x \quad (37)$$

and with eq. (29)

$$-\hbar^2 \Delta_t \psi = E_p^2 \psi \quad (38)$$

and $v \ll c$ and a conservative potential U

$$E_{kin} \approx \frac{1}{2 m_o} p^2 = \frac{E_p^2}{2 E_o} \quad \text{and} \quad E_G = E_{kin} + U \quad (39)$$

we obtain the space independent non relativistic differential equation

$$\left\{ -\frac{\hbar^2}{2 E_o} \Delta_t + U \right\} \psi \approx E_G \psi \quad (40)$$

which is equivalent to the time independent equation from Schroedinger.

5 Uncertainty principle.

In the proposed model the pairs of canonical conjugated variables lead to the following uncertainty relations

$$(\Delta E) \cdot (\Delta x) \gg \frac{1}{2} \hbar c \quad (41)$$

and

$$(\Delta p) \cdot (\Delta t) \gg \frac{1}{2} \frac{\hbar}{c} \quad (42)$$

Noticeable at this point is the relation

$$E r_o = h c \quad (43)$$

for a particle, that connects the radius r_o and the relativistic energy E through $h c$.

6 Operators.

6.1 Relativistic operators.

6.1.1 Relativistic operator for the linear momentum.

The relativistic operator for the linear momentum of a particle is

$$\hat{p} = i \frac{\hbar}{c} \frac{\partial}{\partial t} \quad (44)$$

The linear momentum we get with

$$\bar{p} \chi = i \frac{\hbar}{c} \nabla_t \chi \quad (45)$$

where χ is the total mass-probability function

$$\chi = \psi_x \psi_y \psi_z \quad (46)$$

and ∇_t

$$\nabla_t = \frac{\partial}{\partial t}|_x \mathbf{e}_x + \frac{\partial}{\partial t}|_y \mathbf{e}_y + \frac{\partial}{\partial t}|_z \mathbf{e}_z \quad (47)$$

6.1.2 Relativistic operators for the energy.

For the total energy of a non-accelerated particle we obtain the operator

$$\hat{E}_{tot_x} = -i \hbar c \frac{\partial}{\partial x} \quad (48)$$

Application example.

If we apply the relativistic operators to the relativistic energy of a particle

$$E_x^2 = m_o^2 c^4 + p_x^2 c^2 \quad (49)$$

we get

$$-\hbar^2 c^2 \frac{\partial^2}{\partial x^2} \psi_x = m_o^2 c^4 \psi_x - \hbar^2 \frac{\partial^2}{\partial t^2} \psi_x \quad (50)$$

the **Klein-Gordon** equation.

With $m_o = 0$ we have

$$\frac{\partial^2}{\partial x^2} \psi_x = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \psi_x \quad (51)$$

6.2 Non-relativistic operators.

6.2.1 Non-relativistic operator for the kinetic energy.

The non-relativistic operator for the kinetic energy on the x coordinate is

$$\hat{E}_{kin_x} = -\frac{\hbar^2}{2 m_o c^2} \frac{\partial^2}{\partial t^2} \Big|_x \quad (52)$$

and the total kinetic energy E_{kin} in the three dimensional space

$$E_{kin} = E_{kin_x} + E_{kin_y} + E_{kin_z} = -\frac{\hbar^2}{2 m_o c^2} \Delta_{\mathbf{t}} \chi \quad (53)$$

with

$$\Delta_{\mathbf{t}} = \frac{\partial^2}{\partial t^2} \Big|_x + \frac{\partial^2}{\partial t^2} \Big|_y + \frac{\partial^2}{\partial t^2} \Big|_z \quad (54)$$

6.2.2 Non-relativistic Hamilton operator.

The operator for the non-relativistic total energy on the x coordinate has the form

$$\hat{E}_x = \frac{1}{2 m_o} \left(i \frac{\hbar}{c} \frac{\partial}{\partial t} \Big|_x \right)^2 + \hat{U}_x \quad (55)$$

or

$$\hat{E}_x = \frac{\hat{p}_x^2}{2 m_o} + \hat{U}_x \quad (56)$$

which is equal to the Hamilton operator \hat{H}_x .

The general non-relativistic differential equation thus takes the form

$$i \hbar c \frac{\partial}{\partial x} \psi_x(x, t) = \hat{H}_x \psi_x(x, t) \quad (57)$$

with

$$\hat{H}_x = \frac{\hat{p}_x^2}{2 m_o} + \hat{U}_x \quad (58)$$

the non-relativistic Hamilton operator.

6.2.3 Non-relativistic operator for the orbital-angular-momentum.

The linear momentum operator is

$$\hat{p} = i \frac{\hbar}{c} \frac{\partial}{\partial t} \quad (59)$$

The orbital-angular-momentum-operator can be expressed as

$$\mathbf{M} \left(\mathbf{r}, i \frac{\hbar}{\pi c} \nabla_{\mathbf{t}} \right) = \left(\mathbf{r} \times i \frac{\hbar}{\pi c} \nabla_{\mathbf{t}} \right) \quad (60)$$

with

$$\nabla_{\mathbf{t}} = \frac{\partial}{\partial t}|_x \mathbf{e}_x + \frac{\partial}{\partial t}|_y \mathbf{e}_y + \frac{\partial}{\partial t}|_z \mathbf{e}_z \quad (61)$$

7 The proposed theory and the Correspondence Principle.

The present theory is based on the radius-energy relation derived in [4], relation that substitutes the de Broglie wavelength.

The accordance of the proposed theory with the correspondence principle of the older quantum mechanics is ensured, in that the time independent differential equation from Schroedinger, deduced from the wave package constructed with the de Broglie wavelength, can be derived from the wave package constructed with the radius-energy relation presented in [4].

We start derivating the wave function ψ_x two times versus space, to get the time independent differential equation

$$\frac{\partial^2}{\partial x^2} \psi_x = - \frac{c^2}{\hbar^2} m_{tot_x}^2 \psi_x \quad (62)$$

With

$$m_{tot_x} = \frac{1}{c^2} \sqrt{E_o^2 + E_{p_x}^2} \quad E_o = m_o c^2 \quad \text{and} \quad E_{p_x} = p_x c \quad (63)$$

we get

$$\frac{\partial^2}{\partial x^2} \psi_x = - \frac{1}{\hbar^2 c^2} (E_o^2 + E_{p_x}^2) \psi_x \quad (64)$$

For non-relativistic velocities $v \ll c$ we have that

$$E_{kin_x} = \frac{p_x^2}{2 m_o} \quad \text{and} \quad E_{p_x}^2 = p_x^2 c^2 = 2 m_o c^2 E_{kin_x} \quad (65)$$

and we get

$$\frac{\partial^2}{\partial x^2} \psi_x = - \frac{2 m_o}{\hbar^2} \left[\frac{1}{2} E_o + E_{kin_x} \right] \psi_x \quad (66)$$

With a conservative potential $E_{G_x} = U_x + E_{kin_x}$ we get finally

$$\left[- \frac{\hbar^2}{2 m_o} \frac{\partial^2}{\partial x^2} + U_x \right] \psi_x = E_x \psi_x \quad \text{with} \quad E_x = \frac{1}{2} [E_o + 2 E_{G_x}] \quad (67)$$

For the three dimensional space we have

$$\left[- \frac{\hbar^2}{2 m_o} \Delta_{\mathbf{r}} + U \right] \chi = E \chi \quad (68)$$

with $\Delta_{\mathbf{r}}$ the Laplace operator. Eq. (68) is exactly the time independent differential equation constructed by **Schroedinger** with

$$E = \frac{1}{2} [E_o + 2 E_G] \quad (69)$$

the Eigenvalue.

8 The mass conservation equation.

The mass conservation differential equation we obtain by derivating ψ_x one time versus t and one time versus x . The results are then connected through

$$p_x = \frac{v_x}{c^2} E_{tot_x} \quad (70)$$

We get

$$\frac{\partial}{\partial t} \psi_x(x, t) = - v_x \frac{\partial}{\partial x} \psi_x(x, t) \quad (71)$$

We define the mass probability density as

$$\rho_x(x, t) = \psi_x^*(x, t) \psi_x(x, t) \quad \text{or} \quad \rho(\mathbf{r}, t) = \psi^*(\mathbf{r}, t) \psi(\mathbf{r}, t) \quad (72)$$

We derive the mass probability density versus time

$$\frac{\partial}{\partial t} \rho_x(x, t) = \frac{\partial}{\partial t} [\psi_x^*(x, t) \psi_x(x, t)] = \frac{\partial}{\partial t} \psi_x^*(x, t) \psi_x(x, t) + \psi_x^*(x, t) \frac{\partial}{\partial t} \psi_x(x, t) \quad (73)$$

With eq. (71) we get

$$\frac{\partial}{\partial t} \rho_x(x, t) = -v_x \left[\frac{\partial}{\partial x} \psi_x^*(x, t) \psi_x(x, t) + \psi_x^*(x, t) \frac{\partial}{\partial x} \psi_x(x, t) \right] \quad (74)$$

or

$$\frac{\partial}{\partial t} \rho_x(x, t) = -v_x \frac{\partial}{\partial x} [\psi_x^*(x, t) \psi_x(x, t)] = -\frac{\partial}{\partial x} [v_x \rho_x(x, t)] = -\frac{\partial}{\partial x} j(x, t) \quad (75)$$

or

$$\frac{\partial}{\partial t} \rho(\mathbf{r}, t) = -\nabla_{\mathbf{r}} \mathbf{j}(\mathbf{r}, t) \quad \text{with} \quad \mathbf{j}(\mathbf{r}, t) = \mathbf{v} \psi^*(\mathbf{r}, t) \psi(\mathbf{r}, t) \quad (76)$$

where $\mathbf{j}(\mathbf{r}, t)$ is the mass-current probability density.

9 Conclusions

The fact that it is possible to derive with the proposed theory the time independent **Schroedinger** differential equation (68), ensures that the proposed approach is in accordance with the correspondence principle of the older quantum mechanics.

Accordingly, results of solving the time independent **Schroedinger** differential equation (eg. the harmonic oscillator, the radial Coulomb potential or hydrogen atom, the movement of a relativistic particle deduced by Dirac, etc.), are also valid for the proposed theory.

Also the fact that the space independent non relativistic differential equation (32) derives two times versus time emphasizes its equivalence with the fundamental equation of classical physics, namely Newton's equation.

The following main characteristics were identified,

- The uncertainty principle of Heisenberg between canonical conjugated variables links the pairs **energy-space** and **momentum-time**.

- Because of the changed canonical conjugated variables, the relativistic and non-relativistic operators are reversed respect to time and space derivations compared with the current theory.
- The Schrödinger equation results as the particular time independent case of a more general wave differential equation where the wave function is differentiated two times towards time and one towards space.
- By applying the relativistic operators to the relativistic energy of a particle the **Klein-Gordon** differential equation results.
- The **Hamilton** operator remains formally unchanged.

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